Valuation of complex financial instruments for credit risk transfer

Alfred Hamerle, Andreas Igl

1 Introduction

The fair valuation of complex financial products for credit risk transfer (CRT) can provide a good basis for sustained growth of these markets and their recovery after the current financial crisis. Therefore, the risks of these structured credit securities (such as Collateralized Debt Obligations (CDO) and Credit Default Swap-Index tranches) have to be known as well as the investor's current risk aversion.

Many (even sophisticated) investors rely solely on agencies' ratings for the risk assessment and the valuation of CRT-products due to an information asymmetry between the originators and them. The use of an identical rating scale both for structured products like tranches and corporate securities like bonds tempted many investors to apply identical risk profiles to all products with identical ratings. However, the risk characteristics of CDO tranches differ significantly from comparably rated corporate bonds in relation to systematic risk. Additionally, investors assign different prices to equal cash-flows depending on their risk aversions. Due to the high marginal utility of cash-flows in bad economic times these should have higher weights in a risk valuation approach than income in a benign market environment.

In this article we focus our study on the quite liquid and transparent market of the CDS-Index "iTraxx Europe" and related tranches. We compare market spreads of the tranches with spreads obtained from (I) a simple valuation model integrating the systematic risk sensitivity of tranches and (II) an extended valuation model additionally integrating the investor's risk aversion. Based on our economical reasoning valuation models we obtain significantly differing prices for the investigated complex financial instruments for CRT compared to the market quotations.

Prof. Dr. Alfred Hamerle, Andreas Igl
Department of Statistics, Faculty of Business Management, Economics and Management Information Systems, University of Regensburg, 93040 Regensburg, Germany, e-mail: alfred.hamerle@wiwi.uni-regensburg.de, andreas.igl@wiwi.uni-regensburg.de. We thank Mr. Dr. Liebig and Mrs. Mrasek (*Deutsche Bundesbank*) as well as Mr. Gruber, Mr. Fuechsl and Mrs. Hu (*Bayerische Landesbank*) for providing the data sets within a research cooperation. Working Paper.

2 A framework for credit modeling

Collateral pools of complex financial instruments for CRT are composed of simple assets such as loans, bonds or CDS contracts. Their default behaviour forms the basis for the risk characteristics of multi-name derivatives such as CDOs, CDS-Indices and STCDO. In this paper the default behaviour of the collateral pool's assets follows a Gaussian single risk factor model[1].

Rating-based risk measurement of the collateral pool and the tranches

CRT-products transform the credit risk of the underlying collateral pool into a set of securities with different risk profiles by using credit enhancements like subordination. These tranches represent discriminative parts of the asset pool's loss distribution. A specific tranche incurs losses only if the loss of the collateral pool exceeds the respective subordination level (attachment point). A loss realisation of the asset pool higher than the upper limit (detachment point) of the tranche leads to its total wipe-out.

The rating-based risk measurement of assets in the collateral pool as well as related structured products like tranches may depend on their unconditional expected loss. Using an identical rating scale for both, many investors were tempted to apply similar risk profiles to all products with identical rating grades. Extensive simulation studies show significantly different risk characteristics of CDO tranches in relation to systematic risk (e.g. a macroeconomic factor) compared to corporate bonds with identical unconditional expected loss and therefore equal rating[2]. Figure 1 compares the conditional expected loss "profiles" (conditional upon some systematic market factor M; $\mathbb{E}[L_{tr}|M]$) of a mezzanine tranche with a corporate bond, both with equal expected (loss) rating.

The analysis of conditional expected loss "profiles" (CEL curve) clearly points out that structured products (tranches) react much more sensitively to changes in the macroeconomic factor. Given a critical range of systematic risk factor realisations, $M \in [-6;3]$, the curve of the tranche rises much more steeply than for a corporate bond with comparable rating. The differing impact of systematic risk on financial products leads to consequences both in risk management and in risk valuation.

A "bond representation" approach for structured instruments

In the "bond representation" approach we consider the structured instruments as single-name credit products such as bonds. Therefore, we fit the risk parameters of the "virtual" bond by using the single risk factor model in order to achieve a

---

[1] The derivation of this model from a Merton asset-value model is described e.g. in [6]. The basic asset-value model relates to the findings of Merton in [8].

[2] Extensive simulation studies were performed e.g. in [5] and [6].

preferably good approximation of the tranche's real default behaviour (CEL curve) as well as a good conformity of all risk characteristics.

For the approximation we assume a constant $LGD_{tr}^{(b)}$. The expected loss profile in the bond model is then given by

$$\mathbb{E}\left[L_{tr}^{(b)}|M\right] = p_{tr}^{(b)}(M) \cdot LGD_{tr}^{(b)} = \Phi\left(\frac{c_{tr}^{(b)} - \sqrt{\rho_{tr}^{(b)}} \cdot M}{\sqrt{1-\rho_{tr}^{(b)}}}\right) \cdot LGD_{tr}^{(b)}. \tag{1}$$

For all non-senior tranches (with detachment point $<$ maximum loss in the collateral pool) we set $LGD_{tr}^{(b)} = 1$. We adapt the threshold $c_{tr}^{(b)}$ to ensure that the unconditional expected loss of the bond model equals the simulated unconditional expected loss of the tranche, $\mathbb{E}\left[L_{tr}^{(b)}\right] = \mathbb{E}\left[L_{tr}\right]$.

Furthermore, a nonlinear least squares method between the realized $\mathbb{E}\left[L_{tr}|M\right]$ and the approximated $\mathbb{E}\left[L_{tr}^{(b)}|M\right]$ is used as the fitting criteria of our "bond representation" approach. The estimated value for $\rho_{tr}^{(b)}$ equals the (implied) asset correlation of the tranche and measures its sensitivity concerning changes of the systematic risk factor.

3 Valuation

(I) A simple pricing model based on the CAPM and a Merton asset-value model

Based on the results of the tranche systematic risk measurement, $\rho_{tr}^{(b)}$, (unconditional) risk-neutral default probabilities of a tranche used for valuation can also be derived from a Merton asset-value model. The (unconditional) risk-neutral default probability of a tranche is given by

$$q_{tr}^{(b)} = \Phi\left(c_{tr}^{(b)} + \sqrt{\rho_{tr}^{(b)}} \cdot \delta \cdot \sqrt{T}\right) \tag{2}$$

where δ denotes the Sharpe ratio of the market[3] and T stands for the maturity of the financial instrument. Assuming a zero bond structure, the price of a bond (respectively an approximated tranche) with nominal $EAD = 1$ can be calculated as

$$B^d(0,T) = \exp^{-r \cdot T} \cdot \left(1 - \left(q_{tr}^{(b)} \cdot LGD_{tr}^{(b)}\right)\right). \tag{3}$$

Here, r is the risk-free interest rate. Using the bond price the credit spread s follows from

[3] See [9] for an explanation of the Capital Asset Pricing Model (CAPM) and the Sharpe ratio of the market.

$$s = -\frac{\ln B^d(0,T)}{T} - r. \tag{4}$$

Because of the market Sharpe ratio this valuation approach is based on the assumption of a constant at-the-money implied volatility.

(II) A pricing model using option-implied risk neutral density functions

In contrast, our second valuation approach also integrates the current risk aversion of investors. Based on the Arrow/Debreu theory[4], state prices observed on benign markets should be, due to their low marginal utility, far smaller than state prices observed on stressed markets. Therefore, we use the "DJ EURO STOXX 50" index as a proxy for the market factor M in order to derive state prices of different states of the economy.

On the basis of (daily) market index option prices the "volatility skew" is taken into account and a (daily) risk neutral density (RND) of M is deduced by means of the findings of Breeden and Litzenberger ([2]). The resulting implied risk-neutral density function is then included into the pricing of the CRT-products by

$$B^d(0,T) = \exp^{-r \cdot T} \cdot \int_{-\infty}^{\infty} \mathbb{E}\left[L_{tr}^{(b)} | M\right] f^{RND}(M)\, dM \tag{5}$$

where $f^{RND}(M)$ denotes the risk-neutral density function including the volatility skew (and therefore the risk aversion of the investors). In contrast to the normal density shape of an RND without volatility skew, our density has more probability-mass in the range of stressed states of the economy.

4 Data description

Our empirical analysis is based on spreads of the CDS-Index "iTraxx Europe" and the related standardized tranches, all on a daily basis[5]. Initially we focus our study on the "on-the-run" time period of "series 7" before the credit market came under pressure in fall 2007.

The market equity index "DJ EURO STOXX 50" (price index) is used as a proxy for deriving state prices of different states of the economy. For applying valuation approach II we derive the implied RND function from daily over-the-counter quotes of five-year options on the market equity index for a broad moneyness level after calculation of implied volatilities. The five-year euro area yield curve of AAA-rated government bonds is used for the daily risk free interest rate.

[4] See [1] and [4] for an introduction.

[5] Both spread information and the rating implied (physical) probabilities of default are on a consistent time horizon of five years.

5 Empirical results

The empirical results of this paper can be organized in two sections. *First*, we quantify the systematic risk of financial instruments for CRT by using our "bond representation" approach introduced in section 2. Table 1 shows key statistics of the estimated asset correlation $\rho^{(b)}$ for our time-series, both for the CDS-Index (collateral pool) and the tranches[6].

Table 1 Survey of the estimated asset correlations for the index and tranches by using the "bond representation" approach.

	Tranche width	Mean	Min	Max
Collateral pool	0% - 100%	0.270009	0.269629	0.270170
Tranche 1	0% - 3%	0.671976	0.666913	0.680902
Tranche 2	3% - 6%	0.899765	0.898934	0.901091
Tranche 3	6% - 9%	0.932790	0.931852	0.933808
Tranche 4	9% - 12%	0.946309	0.945451	0.947331
Tranche 5	12% - 22%	0.911069	0.910364	0.912060

It can be seen that the average values of $\bar{\rho}_{tr}^{(b)}$ are much higher for all tranches than those for comparable single-name instruments (e.g. $\bar{\rho}_{Pool}^{(b)} = 0.270009$, estimation using the "bond representation" approach for the collateral pool's assets) reflecting the dramatic increase of the tranche sensitivity to systematic risk[7]. Because of the price relevance of systematic risks the dramatically higher asset correlations of the tranches need to be compensated for by a significantly higher spread.

Second, we compare both the simple valuation model assuming a constant volatility and the extended valuation model integrating the volatility skew as a proxy for the investor's risk aversion with the observed tranche spreads of the market. Figure 2 shows the time-series of the spreads for the CDS-Index as well as for the tranches 1, 2 and 5.

Comparing the tranche spreads of our extended valuation model (dark grey dashed dotted line) with the corresponding market spreads (black solid line) we obtain significantly higher model spreads for all non-equity tranches (tranches 2 to 5). Otherwise, the model spread for the equity tranche (tranche 1) is considerably lower than the market spread. Therefore, our findings are in line with [3].

Moreover, we observe model spreads from the simple valuation approach (light grey dashed line) which hardly differ from the market spreads for the senior tranche

[6] In order to make market spreads and model spreads of the tranches comparable, we calibrate the daily default threshold $c_{Pool}^{(b)}$ of the collateral pool to approximately match the market spreads and the model spreads of the CDS-Index. For the (constant) asset correlation of the collateral pool we use a market standard value of $\rho_{Pool}^{(b)} = 27\%$, see e.g. [7].

[7] As described in [6], the thinner a tranche is the higher is its risk factor sensitivity. Alike the tranche asset correlations rise by increasing the number of assets in the collateral pool or by integrating assets with high systematic risk.

(tranche 5). On the other hand, model spreads for mezzanine tranches are higher than the observed market spreads, but lower than model spreads of the extended valuation approach (tranche 2). As a result of our default threshold $c_{Pool}^{(b)}$ calibration, we find a quite good match of the market index spread and our two model index spreads (Index).

6 Conclusion

Our main findings can be summarized as followed:

- Our outcome indicates that the consideration of the higher systematic risk of tranches as well as the integration of the investors' current risk aversion was different in the market compared to our economical reasoning valuation models.
- Therefore, mezzanine and senior tranches, which react much more sensitively to changes in the market factor than comparable bonds, and which concentrate losses specially in adverse states, require higher risk compensation than offered by the market.
- Otherwise, equity tranches are overcompensated for the taken risk by far too high market spreads.

References

1. Kenneth Joseph Arrow. The role of securities in the optimal allocation of risk-bearing. *Review of Economic Studies*, 31:91–96, 1964.
2. Douglas T. Breeden and Robert H. Litzenberger. Prices of State-contingent Claims Implicit in Option Prices. *Journal of Business*, 51(4):621–651, 1978.
3. Joshua D. Coval, Jakub W. Jurek, and Erik Stafford. Economic Catastrophe Bonds. *American Economic Review*, 99(3):628–666, 2009.
4. Gérard Debreu. *Theory of value: An Axiomatic Analysis of Economic Equilibrium*. Cowles Foundation Monographs Series, 1959.
5. Martin Donhauser. *Risikoanalyse strukturierter Kreditprodukte: Dissertation Universität Regensburg*. 2010.
6. Alfred Hamerle, Thilo Liebig, and Hans-Jochen Schropp. Systematic Risk of CDOs and CDO Arbitrage. *Deutsche Bundesbank, Series 2: Banking and Financial Studies*, (13), 2009.
7. Christoph Kaserer and Tobias Berg. Estimating Equity Premia from CDS Spreads: EFA 2009 Bergen Meetings Paper, 2008.
8. Robert C. Merton. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29(2):449–470, 1974.
9. William F. Sharpe. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, 19(3):425–442, 1964.

Appendix

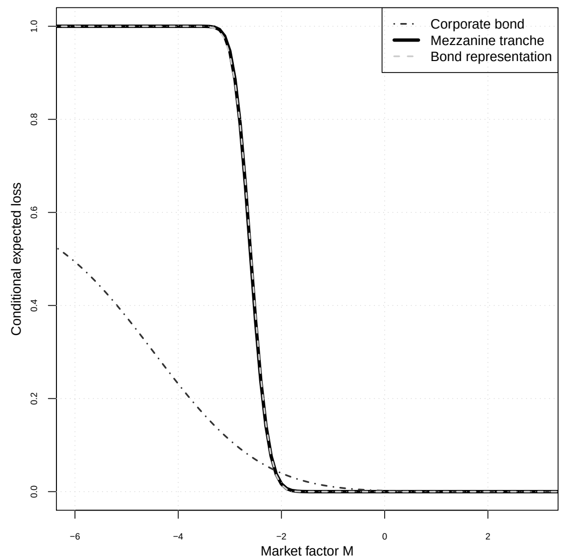

Fig. 1 Conditional expected loss profiles of a mezzanine tranche (black solid line) and a corporate bond (dark grey dashed dotted line) with equal rating. The figure also represents the goodness-of-fit of our "bond representation" approach (light grey dashed line) in contrast to a true CEL curve of a mezzanine tranche (black solid line).

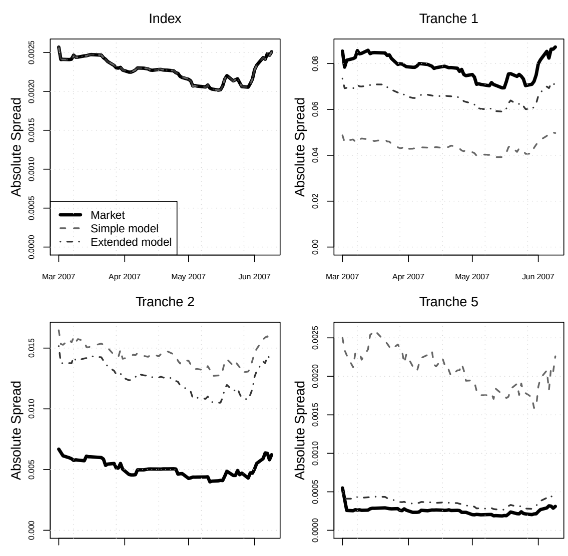

Fig. 2 Market spread (black solid line), model spread of the simple valuation model (light grey dashed line), and model spread of the extended valuation model (dark grey dashed dotted line) compared for the index as well as the tranches 1, 2 and 5.